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RESEARCH PAPER

Elliptical Antenna Array Synthesis Using Backtracking Search Optimisation Algorithm

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ABSTRACT

The design of the elliptical antenna arrays is relatively new research area in the antenna array community. Backtracking search optimisation algorithm (BSA) is employed for the synthesis of elliptical antenna arrays having different number of array elements. For this aim, BSA is used to calculate the optimum angular position and amplitude values of the array elements. BSA is a population-based iterative evolutionary algorithm. The remarkable properties of BSA are that it has a good optimisation performance, simple implementation structure, and few control parameters. The results of BSA are compared with those of self-adaptive differential evolution algorithm, firefly algorithm, biogeography based optimisation algorithm, and genetic algorithm. The results show that BSA can reach better solutions than the compared optimisation algorithms. Iterative performances of BSA are also compared with those of bacterial foraging algorithm and differential search algorithm.

Keywords: Elliptical antenna array, backtracking search optimisation algorithm, sidelobe suppression

1. INTRODUCTION

The antenna arrays are widely used in the defence applications such as communication, radar, sonar, and navigation systems. The main reasons for their common usages are the advantages of antenna arrays over single-antenna structures. Antenna arrays can provide higher levels of gain and directivity. Besides, many modern defence systems are switching from the mechanical steering to the electronic beam steering offered by the antenna arrays. They also have the interference suppression abilities which can be managed by several flexible methods. Subsequently, the antenna arrays are preferred to maximise the signal to noise ratio. In addition to these properties, antenna arrays are capable of estimating the direction of arrival. Because of these attractive features, antenna arrays play an important role in defence and military applications, such as radar, sonar, mobile, wireless, and satellite communications. The performance of the communication systems used in defence systems depends firmly on efficient antenna array design. The pattern of antenna arrays can be shaped by determining the main parameters of the array elements. These main parameters are amplitude, phase and position of the array elements.

Antenna arrays may differ in their geometry. They are generally classified by considering these geometrical differences. Linear, planar, circular, concentric circular, and elliptical antenna arrays are the examples of these groups. The elliptical antenna arrays are relatively new research area. Several methods were used in the synthesis of elliptical antenna arrays[2-8]. An elliptical antenna array was examined in a detailed manner[2]. This elliptical array has equally spaced elements and its main beam was toward to the array normal. The system transfer function was achieved and the formula was given in the paper[2]. New kinds of hybrid antenna arrays were designed by considering the linear and elliptical antenna arrays[3]. The directivities and maximum sidelobe levels (MSLs) were analysed by applying different current distributions. The ellipse eccentricity, array element number, and element spacing were also considered[3,4]. Three different metaheuristic algorithms (SADE, BBO and FA) were used to identify an optimum set of positions for elliptical antenna array[5]. The beamwidth and MSL values were obtained by these techniques and BBO method showed better performance than the other metaheuristic algorithms. Synthesis of elliptical antenna array with low MSL and fixed beamwidth was achieved by using genetic algorithm[6,7]. The array pattern was obtained by the control of the antenna array element phases. The results of genetic algorithm were compared with the results of invasive weeds optimisation. BBO was used to synthesise linear and elliptical arrays[8]. The positions, amplitudes and phases of the antenna array were optimised by using BBO.

Most of the works presented in the antenna array synthesis have concentrated on linear and circular antenna arrays. However, it is well known that the elliptical antenna array has radiation properties similar to those of the circular antenna arrays. Radiation patterns of elliptical antenna arrays cover the entire space; and main beam of the array can be directed to any desired direction. Moreover, compared to rectangular and

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linear antenna arrays, elliptical antenna arrays are less sensitive to mutual coupling since they do not have edge elements[1]. For this reason, in this paper, several elliptical antenna arrays with different number of array elements are considered to achieve optimum radiation patterns by calculating element angular positions and amplitudes with the use of backtracking search optimisation algorithm (BSA)[9]. The antenna array synthesis problems used in this paper consist of MSL and half-power beamwidth (HPBW) constraints. BSA has been able to obtain very good results which meet the constraints. MSL and HPBW of the pattern can easily be controlled by using the BSA. The results of BSA are compared with the results of self-adaptive differential evolution (SADE)[5] algorithm, firefly algorithm (FA)[5], biogeography based optimisation (BBO)[5] algorithm, and genetic algorithm (GA)[8]. BSA shows a better performance in synthesising elliptical antenna arrays than the other compared algorithms. In order to illustrate the computational performance of BSA, the convergence curve of BSA is also compared with the convergence curves of BFA[10] and DSA[11]. It was observed that BSA can converge the better cost function value at the end of the iteration than bacterial foraging algorithm (BFA) and differential search algorithm (DSA). These comparisons indicate that BSA is a good alternative to the other optimisation algorithms for the antenna array synthesis problems. The proposed method is also simple and easy to implement.

BSA is a metaheuristic optimisation algorithm which uses the main evolutionary principles: mutation, crossover, and selection. BSA has been increasingly used for solving different engineering problems owing to its simplicity and high performance. BSA is explained in a detailed manner and compared with several well-known optimisation techniques such as artificial bee colony algorithm, particle swarm optimisation, covariance matrix adaptation evolution strategy, adaptive differential evolution algorithm, comprehensive learning particle swarm optimiser, and self-adaptive differential evolution algorithm[9]. During the comparison tests, 75 boundary-constrained and three real-world benchmark problems were employed. It was concluded that, on the whole, BSA outperformed the compared algorithms. BSA was used for circular antenna array design[12]. BSA was used to synthesis concentric circular antenna arrays and linear antenna arrays[13,14]. Concentric circular antenna arrays with the low MSLs at a fixed beamwidth were designed[13]. Additionally, single, multiple, and broad nulls steering of concentric circular arrays by optimizing only the amplitudes was performed by using BSA. Pattern nulling of linear array using BSA was achieved by computing only the amplitude, position, and phase of antenna array elements[14].

2. PROBLEM FORMULATION

Figure 1 shows the geometry of an elliptical antenna array with N isotropic elements located on x-y plane. When the center of the ellipse is at the origin, the elliptical antenna array factor can be written as[3]:

$$AF(\theta,\varphi) = \sum_{n=1}^{N} I_n \cdot e^{jk\sin\theta(a\cos\varphi_n\cos\varphi + b\sin\varphi_n\sin\varphi)} \tag{1}$$

where I_n is the amplitude of n^{th} array element. N is the number

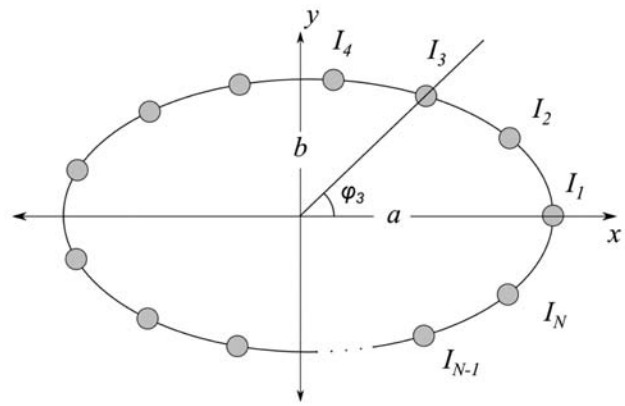

Figure 1. Geometry of an elliptical antenna array.

of elements. and are the semi-major axis a and b semi-minor axis of the ellipse structure, respectively. When θ is set to 90°, φ is the angle between positive section of x-axis and a determined point on the ellipse. On the same x-y plane, φ_n is the angle between the x-axis and the nth element.

Following cost function is used in designing elliptical antenna arrays with the low MSL and fixed or decreased HPBW

$$F_{\cos t} = C_{MSL} \cdot F_{MSL} + C_{HPBW} \cdot F_{HPBW} \tag{2}$$

where C_{MSL} and C_{HPBW} are the weight factors. F_{MSL} and F_{HPBW} are the functions used for suppressing the *MSL* and fixing or decreasing *HPBW*, respectively. The F_{MSL} function can be given as follows,

$$F_{MSL} = \int_{-\pi}^{\theta_{nl1}} \delta_{MSL}(\theta)\cdot d\theta + \int_{\theta_{nl2}}^{\pi} \delta_{MSL}(\theta)\cdot d\theta \tag{3}$$

where $\theta_{n/1}$ and $\theta_{n/2}$ are the two angles at the first nulls on each side of the main beam. $\delta_{MSL}(\theta)$ can be formulated as follows,

$$\delta_{MSL}(\theta) = \begin{cases} \vartheta_o(\theta) - MSL_d, & \vartheta_o(\theta) > MSL_d \\ 0, & elsewhere \end{cases} \tag{4}$$

where $\vartheta_o(\theta)$ is the array factor in dB and MSL_d is the desired MSL value. The function F_{HPBW} can be given by

$$F_{HPBW} = \begin{cases} HPBW_o - F_{HPBW\,\max}, & HPBW_o > F_{HPBW\,\max} \\ 0, & elsewhere \end{cases} \tag{5}$$

where $HPBW_0$ and $F_{HPBW\max}$ are the *HPBW* value obtained by BSA and the desired maximum *HPBW*, respectively.

3. BACKTRACKING SEARCH OPTIMISATION ALGORITHM

The main stages of BSA are initialisation, selection, mutation, and crossover. BSA is an evolutionary algorithm[9]. However, the selection process is divided into two different processes in BSA: selection-I and selection-II. In the initialisation step, all population members are distributed randomly throughout the solution space. The first values are also assigned to the control parameters in this step. Generating trial-population is the task of the mutation and crossover steps. After these steps, the algorithm continues to run the first procedure after the initialisation. The loop goes on until either the stopping criterion is met or it is reached to the maximum iteration number. The best result which it can be reached after the whole optimisation process is the optimum solution.

Initialisation begins with a uniform distribution of population members. It is performed by using following equation:

$$P_i \sim U(low_j, up_j), \quad i = 1,2,3,...,N \quad and \quad j = 1,2,3,...,D \tag{6}$$

where *Pi*, *U*, *D*, and *N* are the position of the *i*th population member in the solution space, uniform distribution function, problem dimension, and population size, respectively. The lower and upper bounds of solution space are denoted by low_j and up_j.

In the selection-I stage, a historical population which is used for determining the search direction is generated. The historical population initializes with the help of following expression:

$$oldP_{i,j} \sim U(low_j, up_j) \tag{7}$$

In the beginning of the each iteration, *oldP* is determined repeatedly by the following statement:

$$if \quad a < b \quad then \quad oldP := P \,|\, a, b \sim U(0,1) \tag{8}$$

where *a* and *b* are the uniformly distributed random numbers in the range of [0,1]. After this process, the order of the population members in *oldP* is randomly shuffled by using following formula:

$$oldP := permuting(oldP) \tag{9}$$

where the *permuting*( ) symbolizes a random shuffling function.

The mutant identities in BSA are produced by using following function:

$$Mutant = P + F \cdot (oldP - P) \tag{10}$$

where *F* is a real number used to change the amount of the step size.

The crossover process of the BSA is given in Fig. 2 in pseudo code style. In the first part of the crossover stage, mix-rate is used in a loop which scans all the population members (Fig. 2, lines 2-4). In the second part of the crossover stage, it is allowed only one randomly chosen population member in order to mutate in each trial (Fig. 2, line 6). Fig. 3 represents the boundary control mechanism of BSA in a pseudo code style. P_is values are updated in the selection-II stage. In this stage, P_{best} is chosen as new global minimum solution when it is set to a new value than global minimum[9].

```
   Input: Mutant, mix-rate, N and D.
   Output: T: Trial-Population.
0  map(1:N,1:D) = 1
1  if a < b | a,b ~U(0,1)    then
2          for i from 1 to N do
3                  map i,u(1:⌈mixrate·rnd·D⌉) = 0 | u = permuting((1,2,3,……,D))
4          end
5  else
6          for i from 1 to N do, map i,randi(D) = 0, end
7  end
8  T := Mutant
9  for i from 1 to N do
10         for j from 1 to D do
11                 if map i,j = 1 then T i,j := P i,j
12         end
13 end
```

Figure 2. Crossover strategy of BSA.

```
Input: T, Search space limits
Output: T
for i from 1 to N do
    for j from 1 to D do
        if (T i,j < low j) or (T i,j > up j) then
            T i,j = rnd · (up j − low j) + low j
        end
    end
end
```

Figure 3. Boundary control mechanism of BSA.

4. NUMERICAL RESULTS

All simulations are carried on a computer which has 2.8 GHz i7 processor and 4 GB RAM. The programming software used for the implementation of BSA is MATLAB. The BSA parameters; mix-rate and step size amplification are set to 1 and F = 3·rndn, respectively. The population size is fixed to 20. The main purpose for the synthesizing problems is to achieve the lowest MSL values to improve the array pattern quality. For the same purpose, the HPBW value is decreased or fixed to a determined value. In order to obtain these low MSL and HPBW values, the angular position and amplitude values of the elliptical antenna array elements are computed by using BSA. In the first three examples, the pattern synthesis is achieved by controlling only the element positions. In the last example, the pattern synthesis is achieved by controlling only the element amplitudes. The elliptical antenna arrays having 8, 12, and 20 elements are considered.

For the first example, the angular position values of an elliptical array which has 8 elements are determined by BSA. The maximum iteration number is 1200. The radiation pattern achieved by BSA is given in Fig. 4. For a comparison, the patterns obtained by using SADE[5], FA[5], and BBO[5] are also presented in Fig. 4. Table 1 gives the MSL and HPBW results of the patterns obtained by BSA, SADE, FA, and BBO. According to Fig. 4 and Table 1, it can be said that BSA has reached to better MSL and HPBW values than those of SADE

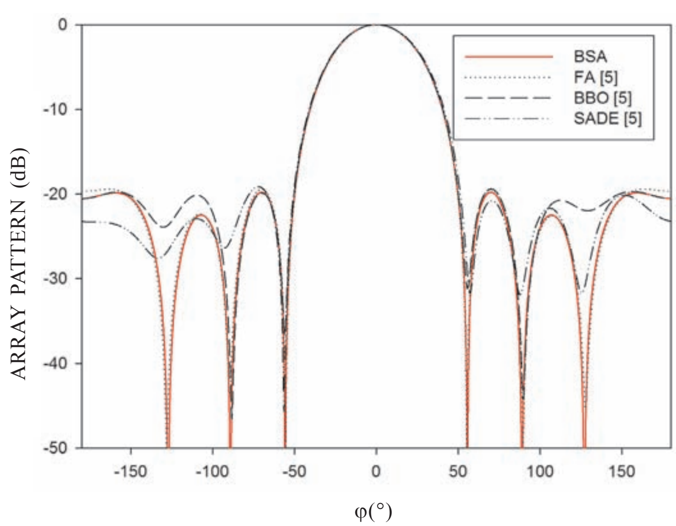

Figure 4. Radiation pattern of 8-elements elliptical array.

Table 1. MSL and HPBW values obtained by BSA, SADE[5], FA[5], and BBO[5] for 8-elements elliptical array

	BSA (Fig. 4)	SADE[5]	FA[5]	BBO[5]
MSL (dB)	-19.78	-19.12	-19.42	-19.40
HPBW (degree)	49.2	50.3	49.2	49.4

and BBO. In Table 1, HPBW values obtained by BSA and FA are the same but MSL value of the pattern achieved by BSA is better than that of FA[5].

An elliptical antenna array with 12 elements is considered for the second example. For this example, 1300 is a sufficient value for the maximum iteration number of BSA. Figure 5 illustrates the radiation patterns achieved by BSA, BBO, FA, and SADE. Table 2 tabulates the MSL and HPBW values calculated by BSA, BBO, FA, and SADE. Figure 5 and Table 2 reveal that BSA can obtain better MSL and HPBW values than those of BBO, FA, and SADE.

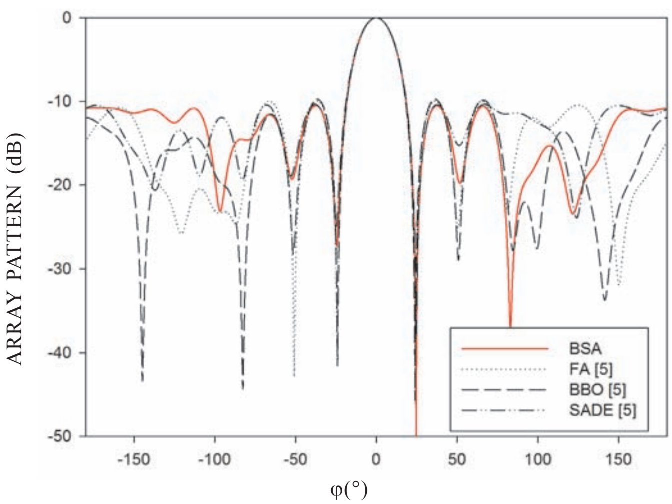

Figure 5. Radiation pattern of 12-elements elliptical array.

Table 2. MSL and HPBW values obtained by BSA, BBO[5], FA[5], and SADE[5] for 12-elements elliptical array

	BSA (Fig. 5)	BBO[5]	FA[5]	SADE[5]
MSL (dB)	-10.50	-9.76	-9.97	-10.37
HPBW (°)	22.0	22.1	22.3	22.5

The elliptical antenna array considered in the third example has 20 elements. The maximum iteration number is set to 1400 for the optimisation process. The radiation patterns plotted with the using the results obtained by BSA, BBO, FA, and SADE are given in Fig. 6. The MSL and HPBW values achieved by BSA, BBO, FA, and SADE are exhibited in Table 3. Both Fig. 6 and Table 3 indicate that BSA has the ability to achieve better MSL and HPBW values than BBO, FA, and SADE. Table 4 tabulates the angular element position values of the elliptical antenna array with 8, 12, and 20 elements.

To show the performance and the flexibility of the proposed method, in the last example, a 12-element elliptical antenna array is optimised by controlling only the amplitude of each array element. The resulting pattern is as shown in Fig. 7. For a comparison, the pattern obtained by GA[8] is shown in Fig. 7. In Table 5, the MSL and HPBW of the patterns obtained

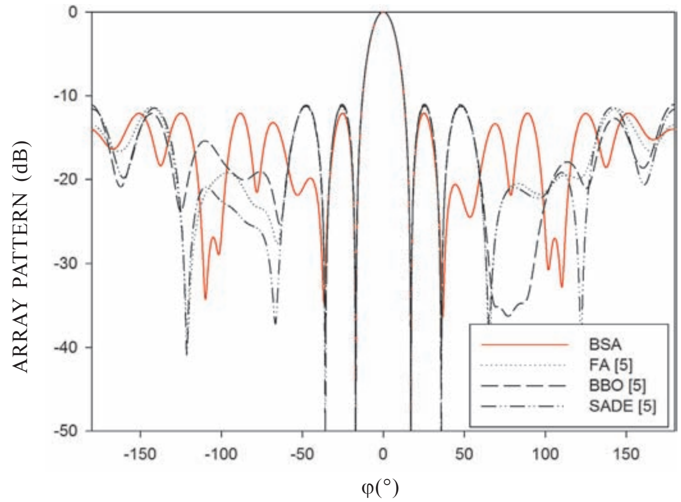

Figure 6. Radiation pattern of 20-elements elliptical array.

Table 3. MSL and HPBW values obtained by BSA, BBO[5], FA[5], and SADE[5] for 20-elements elliptical array

	BSA (Fig. 6)	BBO[5]	FA[5]	SADE[5]
MSL (dB)	-12.09	-11.02	-11.27	-11.23
HPBW (degree)	15.2	15.3	15.5	15.4

Table 4. The angular element positions of elliptical antenna arrays

N (Element Number)	[$\varphi_1, \varphi_2, \varphi_3, \ldots \varphi_N$] (degree)
8 (Fig. 4)	[30.82, 53.35, 126.65, 149.18, 210.82, 233.35, 306.65, 329.18]
12 (Fig. 5)	[18.77, 19.80, 76.94, 88.45, 159.07, 168.21, 200.96, 258.79, 273.99, 296.42, 339.63, 358.82]
20 (Fig. 6)	[4.52, 8.84, 44.00, 81.42, 90.24, 92.58, 107.00, 139.65, 172.27, 180.47, 181.68, 184.94, 221.17, 253.14, 268.36, 268.99, 278.67, 316.43, 350.86, 356.77]

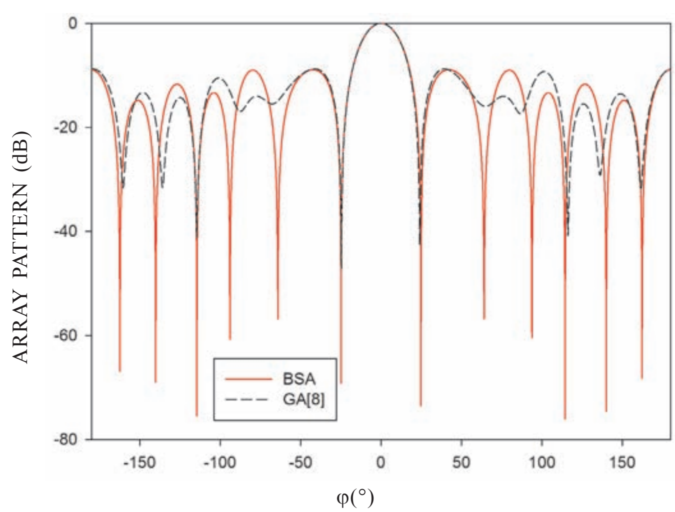

Figure 7. Radiation pattern obtained by amplitude-only control for 12-elements elliptical array.

by using BSA and GA[8] are given. Table 5 shows that MSL and HPBW of the pattern produced by using BSA are better than those of GA[8]. The amplitude values computed by BSA for the pattern illustrated in Fig. 7 is given in Table 6. It is seen from Table 6 and reference 8 that dynamic range ratio ($DRR=|I_{max}/I_{min}|$) of the pattern obtained by using BSA is better than that of GA[8]. Overall, according to the obtained results, proposed algorithm BSA can achieve better results than FA[5], BBO[5], SADE[5], and GA[8].

Table 5. MSL and HPBW values obtained by BSA and GA[8] for 12-elements elliptical array

Algorithm	MSL (dB)	HPBW (°)
BSA	-8.92	22.2
GA [8]	-8.75	22.3

Table 6. The amplitude values of 12-elements elliptical antenna array (Fig. 7)

$I_1, I_2, I_3, \ldots I_{12}$	[0.9617, 0.1485, 0.3390, 0.4636, 0.3391, 0.1481, 0.9614, 0.1445, 0.3430, 0.4614, 0.3429, 0.1449]

To show the computational performance of BSA, the convergence curve of BSA is compared with the convergence curves of BFA[10] and DSA[11] in Fig. 8 for the first example. The population number and the maximum iteration number for all three algorithms are 50 and 1200, respectively. The parameters of BFA are set as follows: length of swim is 4, number of chemotactic steps is 300, number of elimination-dispersal events is 1, number of reproduction steps is 4, and length of run is 0.4. DSA does not require any extra parameter definition. It is apparent from Fig. 8 that BSA can converge the better cost function value at the end of the iteration than BFA and DSA. In Table 7, the computation times of BSA for the first three examples are given. The computation times given in Table 7 are sufficient to obtain desired patterns.

Table 7. Computation times of BSA for the first three examples

Number of elements	8	12	20
Computation time (s)	68.2513	141.3256	286.5125

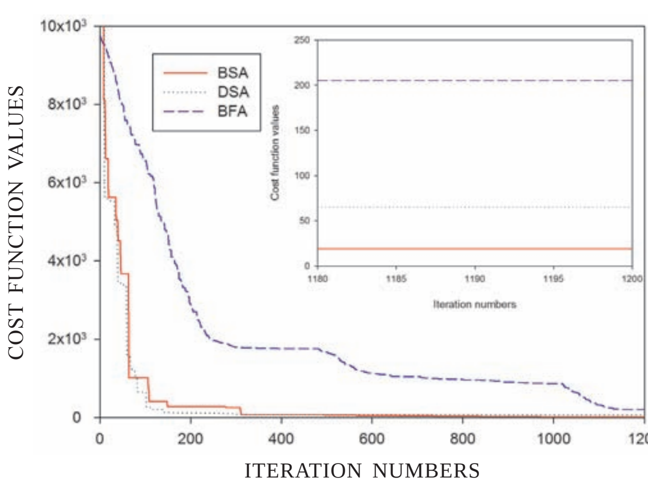

Figure 8. Converge curves of BSA, BFA[10], and DSA[11].

5. CONCLUSIONS

In this paper, elliptical antenna arrays with 8, 12, and 20 isotropic elements are synthesised by using BSA. The lower MSL and narrower HPBW values are two main targets. In order to achieve the desired patterns, the angular position and amplitude values of the antenna array elements are optimally obtained by BSA. The results show that BSA is very successful optimisation algorithm in the synthesis of elliptical antenna arrays. It is also apparent from the results that BSA is better than FA, BBO, SADE, and GA in terms of MSL and HPBW. Furthermore, the convergence curve comparison illustrates that BSA provides better convergence performance than BFA and DSA.

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In the current study, he has contributed in the design of elliptical antenna arrays.

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In the current study, he has contributed in synthesis of elliptical antenna arrays using backtracking search optimisation algorithm.